1 Target Issue: Complex problem solving in very distant spatial tasks.

Experienced submarine Approach Officers (AO) have developed complex skills for mentally turning the alphanumerics of passive sonar into spatial representations of other vessels, their paths, intentions, and the high uncertainty of the undersea world (1997). How they make this translation and what is contained in the spatial representations remains unclear. The translation is one of the most difficult tasks for submariners to learn, and once mastered, it is still a skill that submariners find critical, but complex and subject to error (Kirschenbaum, personal communication).

Not all kinds of space are psychologically the same. Developmental, neuropsychological, and adult behavioral data suggest that different sizes of 3-dimensional space are processed in very different ways (Huttenlocher, Newcombe, & Sandberg, 1994; Previc, 1998; Weatherford, 1982). For example, Previc (1998) distinguishes four major categories of spatial sizes: peripersonal (visuomotor operations in near-body space), focal extrapersonal (visual search and object recognition), action extra-personal (orienting in topographically defined space), and ambient extrapersonal (orienting in earth-fixed space). AOs make use of all of these categories of space, and thus accounts of the complex problem solving in which AOs engage must take into account the properties of each of the spatial grain-size activities. However, it is currently unclear which representation categories are important for a given task. It is also currently unclear what psychological mechanisms and representations implement each category.

The goal of this literature review is to evaluate our current understanding of spatial information is represented, such that this understanding can guide the development of computational models for complex spatial problem solving like in submarine target motion analysis.

2 Framework & Central Questions

Before covering psychological findings regarding representations of human visual space, I first present a general framework in the form of common distinctions and central questions about representations of human visual space.

2.1 Egocentric vs. Exocentric Frames of Reference

One of the most basic questions about space is the frame of reference or coordinate system in to which all objects and points are defined in relative terms. Two primary frames of reference are generally distinguished and go by the terms egocentric and exocentric. Egocentric frames of reference use the observer as the center of reference with their orientation as the referential axis of orientation. Exocentric frames of reference use the some other point or some other axis of orientation defined outside of the observer as the frame of reference (e.g., the earth or the main axis of a room).

One can always convert from a location in one frame of reference to a location in another frame of reference (e.g., from ego to exocentric). But, this distinction is important because the two types of frames of reference appear to have different pieces of primitive information which are automatically and directly represented (Klatzky, 1998), which can have a large influence on performance.

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14. ABSTRACT
The nature of human visual-spatial representations have been studied from a variety of disciplines, and have contributed to a rich understanding of how humans process space. This report presents a summary of the literature, including work from computational cognitive science, cognitive neuroscience, mathematical psychology, developmental psychology, and cognitive psychology. The literature highlights the core dimensions that emerged: types of representations, biases in representation, dimensionality of representation, and multiplicity of representation.

15. SUBJECT TERMS
spatial cognition; representation; complex problem solving

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2.2 Types of Spatial Knowledge

In representing space, three main types of information can be represented. The first is landmark information. Landmarks are familiar objects that serve as isolated reference points in our spatial knowledge. These landmarks can be used in conjunction with egocentric frames of reference (e.g., turn left when you get to a fork in the road) or with exocentric frames of reference (e.g., turn south when you get to a fork in the road).

The second type of knowledge is route knowledge. Routes are paths through a space and include some topological information and some metric information. Route knowledge is likely to have declarative components (direct representations of space) and procedural components (knowledge of what to do in different regions). Routes can also be defined either in ego or exocentric terms (Hunt & Waller, 1999). Use of local, exocentric route cues is called tracking (e.g., following highway signs). Use of egocentric turns and distances is called dead reckoning (e.g., go 3 paces straight ahead). Integration of egocentric bearings with exocentric locations is called piloting (e.g., go 1 mile straight past the 2nd light).

The third type of knowledge is configuration information and consists of a global, metric map of a space. Configuration knowledge can also be egocentric or exocentric. Configuration knowledge is the most complete representation of a space and is usually the most difficult to maintain (Linberg & Garling, 1983) and last to develop (Foley & Cohen, 1984; Siegel & White, 1975).

2.3 Symbols versus images

A long-standing debate in cognitive science surrounds that format of internal representations: is it represented symbolically or perceptually? Some researchers argued that all representations are inherently symbolic whether perceptual-like or not (Vera & Simon, 1993). Others argued that the two are informationally equivalent and that we will never be able to distinguish the two empirically (Pylyshyn, 1989). However, neuropsychological evidence suggested that many representations are indeed perceptual (Kosslyn, 1990). More recently, Barsalou (1999) has argued that even abstract, conceptual entities may have a perceptual representation.

Very much related to this issue is the type of scale included in a representation of space. Four different scales are usually distinguished: nominal, ordinal, interval, and ratio. Nominal scales represent entities in terms of unordered categories (e.g., left/right or locations A, B, or C). Ordinal scales represent entities in terms of ordered categories (e.g., left/middle/right or here/close-by/faraway or above/my-level/below). Interval scales represent entities in terms of metric amounts (i.e., a given difference has the same meaning across the scale), but there is no real zero point (i.e., the ratio of absolute locations has no meaning). One might argue that exocentric coordinates involve essentially interval scales. By contrast, ratio scales represent entities in metric terms with a meaningful zero (e.g., distance in meters of an object relative to the observer).

2.4 One or Many Representations

A central question in studies of visual space is whether there are one or many representations. This question decomposes into questions about whether there are many different formats, different processes, and different locations in the brain. This issue is of particular relevance to this proposal because it turns out that there are indeed many different representations and they appear to vary with the scale of the space being represented. Since

complex spatial tasks like the one described at the beginning of the background section involve interacting with space at multiple scales, it becomes especially important to consider multiple, different spatial representations.

3. Properties of Human Visual Space

Visual space as perceived and retrieved from memory by humans is not a simple reconstruction of Euclidean space. First, the deviations from Euclidean space are systematic and fall into several different categories. Second, it appears that there are several different representations of space that people use. Third, it appears the representations within an individual fluctuated over time. This section describes each of these results and the empirical evidence from the cognitive, neuro, mathematical, and developmental psychology supporting them.

The developmental literature is applied to the cognition of adults in the following way. It is assumed that two things are likely to be true of two abilities that develop at different times developmentally. First, the later developments are likely to indicate different underlying process or the same processes working on different representations. Second, it is likely that later occurring developments involve functions that continue to be processed less readily in adults than those associated with early occurring developments, for reasons of task expertise and sophistication of required representation. Abilities that have developed later will, by definition, have had fewer opportunities to be practiced and thus are likely to have been practiced less and so are less automated. Functions that involve the same processes but more sophisticated representations will also be continued to be processed more slowly.

3.1 Biases in Representation

3.1.1. Curvature in the horizontal plane

The visual space that we perceive has systematic curvature in the horizontal plane that is hyperbolic in form (Indow, 1991, 1997; Luneburg, 1947, 1950). This hyperbolic bending of space has two features. First, visual space that is relatively near to us appears concave (bent towards us) and space that is relatively far from us appears convex (bent away from us). Second, lines that appear to be straight out into the distance are actually bent outwards. This hyperbolic bending of space applies only to horizontal planes extending out from our eye-level (either flat or tilted). Frontoparallel planes of space (i.e., planes in front of us parallel to our bodies) appear to have regular Euclidean geometry (or at least do not have this type of curvature).

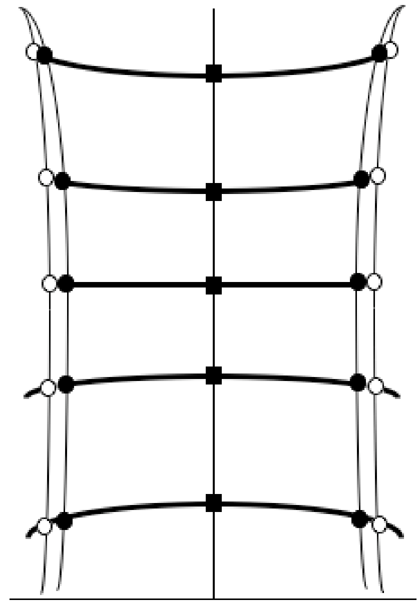

Figure 1. Series of points on a flat horizontal plane extending from the observer that appear to be in parallel lines (filled circles) or equidistant points (open circles). Filled in squares are the fixed points used in horopter experiments, and the horizontal curves extending through them represent H-curves. Vertical curves are theoretical curves representing a hyperbolic model of visual space fit to the data.

The first piece of evidence for the hyperbolic bending of space comes from alley experiments. In alley experiments, participants make decisions about pairs of points in a frameless visual space that is achieved either through points of light in a dark room or small objects in an evenly illuminated surface with invisible edges. Participants adjust the points according to two different criteria to form P-alleys and D-alleys. For P-alleys, participants are shown pairs of points extending along the y-axis (away from the participant) and are asked to adjust them along the x-axis (left-right axis) until they appear to form straight and parallel lines. For D-alleys, participants are again shown pairs of points extending in the y-axis and are asked to adjust them in the x-axis such that each pair of points has the same lateral separation as that of a fixed, reference pair. Although the two types of alleys produce slightly different results (Blumenfeld, 1913; Indow, 1991), with P-alleys generally lying inside D-alleys, both sets of alleys diverge out into the distance, consistent with a hyperbolic bending of space.

The second piece of evidence for the hyperbolic bending of visual space comes from horopter experiments (Luneburg, 1950). In these experiments, participants again make judgments in a frameless visual space. Participants adjust series of points along the y-axis such that they appear to make a straight line that runs left-to-right in parallel to the forehead of the participant. Each series consists of three points, with the center point being fixed in space. The curve through these three points is called an H-curve. This process is repeated for series of points at different y-distances from the observer (see Figure 1). These experiments find that the participants' judgements are systematically biased such that the H-curves are convex at distances far from the observer and convex at distances close to the observer. The crossover point is somewhere around 2 meters away from the observer (Indow, 1991), although the exact crossover point varies across individuals (Luneburg, 1950).

The exact form of the bending is an issue of debate, for example whether the (Gaussian) curvature is constant across all points in space (Eschenburg, 1980; Indow, 1991). The degree of bending has been observed to depend upon the nature of the stimuli. For example, illuminated spaces produce differential bending than dark spaces (Indow, 1997; Indow & Watanabe, 1984). Moreover, more bending was found in experiments performed in large spaces (e.g., a gymnasium) than for experiments performed in small laboratory spaces (Indow, 1991).

3.1.2. Two-dimensional bias towards anchor points

When visual space is examined in the context of other objects (in contrast to frameless visual space) additional biases appear. For example, when people must reproduce the location of a point in two-dimensional space, their estimates for the location of that point is systematically biased towards visual anchor points. In a series of experiments, Huttenlocher, Hedges, and Duncan (1991) found that when asked to reproduce the location of a dot within a circle people show a consistent pattern of bias. In particular, the estimates for dot locations are biased toward the center of mass within each quadrant of the circle (see Figure 2).

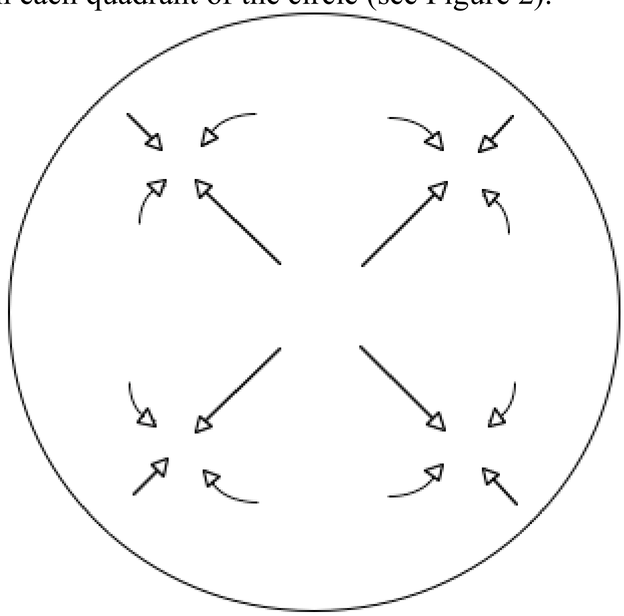

Figure 2. Schematic representation of the bias in reports of location in a circle.

To account for these results, Huttenlocher et al. postulated that memory consists of unbiased, but noisy fine-grain coding of information together with gross-grain categorical information that is combined to produce the biased estimates. Subsequent developmental studies supported this hierarchical decomposition of spatial knowledge. Huttenlocher, Newcombe, and Sandberg (1994) found that 16-month-olds treat objects as whole without subparts when coding spatial location (i.e., show no systematic biases towards anchor points within the object). They also found that children subdivide objects of increasing complexity as they get older. For example, a rectangle is subdivided by 4-year-olds, and a sandbox is subdivided by 10-year-olds (i.e, show systematic biases towards anchor points within the objects). Follow-up work by Sandberg, Huttenlocher, and Newcombe (1996) suggests that the simplified representations used by younger children are likely to be due to cognitive load rather than difficulties in representing particular kinds of information per se.

3.1.3. Differential treatment of geometrical and non-geometrical landmarks

In locating objects and locations in the environment, people use two different types of landmarks. First, they use large-scale geometrical landmarks such as the shape of the terrain (as determined by mountains, valleys, or nearby buildings) in outdoor contexts, and the overall geometric shape of the room in indoor contexts. Second, they use non-geometrical landmarks such as the location of the sun, the direction of the wind, the color of a wall, the patterning of a wall, or the categorical identify of objects in the environment. In re-orienting oneself in an environment, the individual combines these two types of information.

Developmental and comparative psychological research suggests that these two types of information are represented differently and develop at different times. For example, Hermer and Spelke (1994, 1996) found that young children fail to use nongeometric landmarks in reorienting themselves in a room, whereas adults do use such nongeometric information. Adult rats also have such difficulty in using nongeometric landmarks (Margules & Gallistel, 1988). Hermer and Spelke (1996) found that the young children were able to detect, remember, and use the same nongeometric information. They argue that these results suggest that the problem is not one of representation but rather one of information encapsulation: the information is encoded so differently that more sophisticated processes are required to integrate the two types of information to be used in spatial re-orientation.

3.1.4. Influence of perceived vs. imagined reality conflicts on spatial processing

Some spatial reasoning tasks require that we imagine spatial situations other than those in front of us. The developmental literature suggests such a task is especially difficult when the currently perceived reality clashes with the to-be-imagined reality.

Most of the developmental psychology literature on spatial reasoning has focused on the perspective-taking problem, first addressed by Piaget. The common finding is that viewer rotation is harder than array rotation, and children below 9 or 10 cannot solve viewer rotation problems. That is, they cannot answer the question "what would the arrangement of objects look like from the other side of the table?" but they can answer "what would the arrangement of objects look like if the table were rotated?" Piaget and followers claimed that this result is due to egocentrism (Flavell, 1968; Piaget & Inhelder, 1966). In other words, the claim is that children below a certain age cannot imagine a perspective other than their own. However, later work by Presson (1982) showed that the even adult humans process the two types of situations differently.

Huttenlocher and Presson (1979) showed that this difficulty in viewer rotation is not due to egocentrism and the effect can be reversed. For example, they found that eight-year-olds could solve problems asking them to imagine the relative location of an item with respect to a hypothetical observer (in contrast to the traditional question asking about the relative location of items to each other from the perspective of a hypothetical observer). Newcombe and Huttenlocher (1992) extended these results to much younger children and argued that the difficulties in viewer rotation problems is due to conflicts between the currently perceived reality and the reality that must imagined to solve the task. That is, if the two are too similar (requiring similar objects and type of relational encoding), the perceived reality intrudes on the imagined one.

3.1.5. Non-equivalence among spatial directions

When searching through imagined 3-dimensional environments, not all spatial directions are searched equally well. In particular, the left/right direction appears to be searched more slowly than front/back or up/down directions. Franklin and Tversky (1990) had participants

answer questions about objects place in an imagined environment in various directions relative to the participant. Across a variety of experimental manipulations, the left/right dimension was always search more slowly than the other dimensions. Note that left and right refers to objects directly to the side of the observer out of the field of view, not to objects in front of the observer slightly to the left and right that would be searched quite quickly.

A similar slowing of the left/right dimension was found by Hintzman, O'Dell, and Arndt (1981) in a series of (2-dimensional) experiments in which participants had to point to targets while imagining themselves in a particular spot facing in various directions. Newcombe and Huttenlocher (1992) also found that left/right is harder than near/far for children—a sticker was placed on one of the children's hands to make sure the problem was not one of knowing "left" and "right" labels.

Franklin and Tversky interpreted their results in terms of a spatial framework model in which the up/down and front/back dimensions are more functionally important in the environment. They also argued that their results ruled out a spatial transformation account in which the participant mentally rotated in their environment because left/right questions (i.e., at 90 degrees) were answered significantly more slowly than questions about what was behind (i.e., at 180 degrees).

3.2 Multiple Representations of Space

3.2.1 Neuroscience division of multiple representations

From a purely neuroscience perspective, human visual space is incredibly complex. At the process level, vision is a very complex process by which information is combined to understand (perceive and recognize) the 3-dimensional world from essentially 2-dimensional information on the retina. More importantly to this proposal, neuroscientists have long called for multiple spatial representations based on dissociations in localized human brain damage, lesion studies with animals, single-cell recording with animals, and, more recently, brain imaging studies with humans. A variety of proposals have been put forth over the years for the number of representations and their content (Brain, 1941; Cutting & Vishton, 1995; Grusser, 1983; Mountcastle, 1976; Pettigrew & Dreher, 1987; Previc, 1990, 1998; Rizzolatti, Gentilucci, & Matelli, 1985; Rizzolatti, Matelli, & Pavesi, 1983).

The most comprehensive model is that of Previc (1998). This model calls for 4 different representations of space. Each representation corresponds to different spatial domains, going from very close to the individual to very far away, with differential degrees of emphasis towards upper and lower visual fields and degree of coverage around the body. Each representation involves different types of activities/functions of space. Each representation has different functional properties. Finally, each representation has different anatomical localizations in the brain. Here I will review the characteristics of each representation. See Previc (1998) for a detailed review of the neuroscience evidence supporting these distinctions. Although the different spaces also involve senses beyond vision to different degrees, I focus on the relationship to vision here because that is the focus on my proposed research.

The first representation is of peripersonal space and involves visuomotor operations in near-body space (e.g., visual grasping and manipulation). It extends between 0 and 2 meters from the body, in a 60-degree arc in front of the body, with a bias towards the lower visual field. It appears to use egocentric coordinates, primarily body-centered (Gaffron, 1958; Previc, 1990). It appears to be represented primarily in dorsolateral cortex.

The second representation is of focal extrapersonal space and is involved in visual search and object/face recognition. It extends from .2 meters from the body into the distance, in a 25-degree arc in front of the body, with a bias towards the upper visual field. It appears to use egocentric coordinates, most likely using retinotopic coordinates (Deneve & Pouget, 1998; Farah & Buxbaum, 1997). It appears to be represented primarily in ventrolateral cortex.

The third representation is of action extrapersonal space and is involved in navigation (in relation to objects and topographically defined space), scene memory, and target orientation. It extends from 2 meters into the distance, completely around the individual (although with compression outside of 200 degrees), with a bias towards the upper visual field. It appears to use egocentric coordinate, particularly gaze-centered coordinates (Bisiach & Luzzatti, 1978; Gaffan, 1991; Rolls & O'Mara, 1995). It appears to be represented primarily in the ventromedial cortex.

The fourth representation is of ambient extrapersonal space and is involved in spatial orientation, postural control during locomotion, and stabilizing perception of the world during locomotion. It covers region more than 2 meters away from our bodies, in 180-degree arc in front of us, with a bias towards the lower visual field. It appears to use exocentric coordinates with a bias to use gravity and the earth as a frame of reference (Angelaki & Hess, 1995; Patterson *et al.*, 1997). It is represented primarily in the dorsomedial cortex.

The neuroscience literature suggests that representations have clear input/output connections. For example, the focused extrapersonal space is used for controlling saccades. However, what kinds of spatial representations that a person will have at any point in time is a complex issue.

First, tasks often require multiple inputs or multiple outputs. For example, searching for a door in larger room requires involves focused visual search and overall spatial orientation. Thus, one simple task may activate multiple representations.

Second, the spatial representations are used in a coordinated fashion. For example, overall spatial maps of the surrounding area are used to initialize focused visual search. Similarly, in moving to a door and opening it, one first finds the door (visual search), walks to it (navigation), updates ones location relative to the room (localization), and finally grabs the handle and turns it (eye/hand coordination). Information in one representation is in part passed to the other representations. For example, the identity and location of the target object (e.g., the door in the previous example) are shared across representations.

Third, individuals may engage in re-coding from one representation to another even if the current external task does not require it. This possibility is especially likely in complex tasks that require significant cognitive activities (e.g., planning). Because each representation uses different coordinate systems and has different biases, an individual may shift representations depending on what works best for the current cognitive task.

The possibility of multiple simultaneous representations that must be coordinated at times with possible re-coding of information make prediction and analysis of behavior in complex tasks very difficult from verbal theories of the tasks and the representation systems. To really understand how these representations interact so that predictions can be made for particular situations, one must build a computational framework that instantiates the computational and behavioral properties of these representations in a very precise way.

3.2.2 Computational modeling & neuroscience evidence

From a computational modeling perspective, there is a real temptation to take one of two different extreme approaches in response to this complexity. The first extreme approach is to completely ignore the neuroscience evidence and assume a single, computationally convenient

way of representing spatial information. One might argue that ACT-RP/M has taken this approach. There, visual space is represented in terms of objects at precise X-Y locations, without any treatment of bias or orientation issues, without any treatment of a third dimension. The advantage of this extreme approach is that the representation process is easy to understand, easy to implement, and appears to work fine in modeling domains involving flat displays with relatively little spatial reasoning. Finally, as cognitive science has long known, there is a great deal of informational equivalence in representations—with a little bit of work you can represent almost anything in any given type of representation.

The second extreme approach is to take the neuroscience evidence quite literally and build a separate functional box corresponding to each neurologically distinct region. This is the approach taken by connectionist models of space (Touretzky & Redish, 1995, 1996). The advantage of this approach is that each box can be built exactly to specification. Thus, this approach appears to work fine in modeling very simple, representation-targeted tasks—only one functional box is required at a time in such tasks.

Both of these extreme approaches have their perils. The first extreme is clearly going to be wrong whenever the to-be-modeled task involves careful and detailed spatial reasoning. While there is some informational equivalence across representations, there is not computational equivalence. In other words, certain representations afford certain computations (Larkin & Simon, 1987).

The second extreme is going to be of limited use in modeling complex behavior, for several reasons. First, it is computationally very expensive and difficult to implement, and thus not likely to be used in the kinds of situations that complex spatial tasks involve. Second, it places too large a separation between representation and process. Each functional box tends to have its own processes. This approach then tends to miss the more parsimonious approach of different representations with a common set of processes (with common learning mechanisms and common performance mechanisms). Neuroscience evidence has not yet ruled out this intermediate case; Occam's razor should be applied. Indeed, the similarity in learning patterns (e.g., the ubiquitous powerlaws of learning and of forgetting) across a wide variety of tasks is suggestive of a common procedural core. Third, there is little understanding of how the separate functional boxes interact to produce a single behavior at a given point in time, especially in tasks that require using multiple boxes.

For these reasons, I propose to use a variant of the intermediate case: multiple, independent, and potentially simultaneously active spatial representations with different properties, but all accessed by a common core procedural core with a common set of learning and performance mechanisms.

3.2.3 Cognitive psychology of multiple representations

Because of recoding, people can build many possible representations no matter what the input, although there are likely to biases in choice of representation given a particular input. For example, people can develop cognitive maps from moving fingers over map while blindfolded, walking a path blindfolded, or viewing a map (i.e., similar representations from different input modalities) (Levine, Jankovic, & Palij, 1982).

In addition, a given task can result in multiple, simultaneous representations, both because multiple representations are required by the task (e.g., reading requires building structural, phonemic, and semantic representations) and because incidental, automatic processing (e.g., semantics are often processed during non-semantic tasks like lexical decision). Therefore,

it is reasonable to create a model that creates multiple layers of representation and accesses them opportunistically.

Not all representations are equally valuable for spatial problem solving. For example, Hagerty and Kozhevnikov (1999) found that schematic representations but not pictorial representations were positively correlation with solution success in mathematical problem solving. Therefore, it is important to examine the role of each type of representation in problem solving success.

3.3 Variability in Content of Representations

Even assuming a fixed kind of spatial representation format with a given set of perceptual inputs and fixed coordinate system, there is still likely to be considerable variability in the content of the representations. This variability can have a large impact on reasoning that uses this content, and thus is an important, although often overlooked, aspect of representations.

3.3.1 Uncertainty

The first layer of variability stems from uncertainty of two different types: perceptual and conceptual. Perceptual uncertainty stems from the inexact magnitude estimates that the human perceptual system produces (e.g., in estimating distances of objects, their magnitudes, their speed, etc.) Here representations can change simply because of varying perceptual estimates of magnitude change noticeably over time. For example, one might revise ones estimate of the distance of an object several times over a relatively short period.

Conceptual uncertainty stems from an understanding of the misleading or noisy nature of various perceptual inputs. For example, in reading sonar location readings, a submarine approach officer realizes that the apparent depression angle of the target (i.e., the apparent depth of the source of the sounds) may be completely different from the actually depression angle of the target because of the way sound bounces off the ground and is bent through layers in the ocean. Gestures made by the approach officers reveals that they are directly representing this uncertainty: when the discuss objects who location is still open to interpretation, they use vacillating gestures in representing the object (e.g., a quivering hand) to indicate uncertainty about object's location.

3.3.2 Depth of information

The second layer of variability in representations of a fixed format stems from different kinds of dimensional reduction that people use. Work by Shah and Carpenter (1995) suggests that people can vary quite significantly in the type of scale they use to represent a given dimension. That is, even when people are given perceptual input that may be interpreted as a ratio scale, people may sometimes re-represent that information in ordinal, or even nominal terms. This reduction in dimension quality may be in response to the perceptual and conceptual uncertainties in the situation.

Applied to the submarine domain, one sees that approach officers often use nominal representations to represent depth in the water. For example, they think of objects as being either above the water, on the water, somewhere above them, at the same level, or somewhere below them. Similarly, they sometimes think of the x-axis (in the fronto-parallel plane) as being ordinal: to the left, in the center, or to the right.

3.3.3 Variability in represented objects and features over time

The third layer of representational variability is variability in represented objects and features. Recent work (Lovett & Schunn, 1999; Schunn & Klahr, 2000) suggests that even in relatively simple tasks, people may be constantly changing which objects and which object

features they represent, especially when the task is currently difficult. That is, people are constantly search for new things to include and exclude from their representation of the current task. These representation changes can have profound effects on what people can learn from their experiences (Lovett & Schunn, 1999).

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